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Analysing different payment methods in multi-unit auctions: An empirical evidence from Delhi NCR India

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Abstract
This paper aims to provide empirical evidence for the revenue ranking between discriminatory and uniform price auctions by analysing the market for fractional real estate investment. Additionally, it endeavours to reveals the fact related to quantity of biding under discriminatory and uniform price auction based on the primary data of Delhi-NCR. The empirical outcome reveals that we can discard the assumption of equality average quantity of bidding under the discriminatory and the uniform price auction. Additionally, our result clearly reveals that investors do react differently under discriminatory and the uniform price auction, hence it can be surmised that nature of price auction does have significant impact on the average quantity of bidding. On the other hand, the outcomes of F test reveal that variability in biding quantity under two price schemes are dissimilar. Finally, our result exhibit that revenue generated under the uniform price auction is higher then the discriminatory price auction, hence revenue maximization firms should rely on the uniform price auction.

Keywords: Discriminatory price auctions, uniform price auctions, clearing price, winners curse, demand shading

1. Introduction

Multi-unit auctions involve the sale of multiple homogenous items or shares to a set of bidders. These auctions have become increasingly popular due to lower prices, increased accessibility, and lower risk, and are used in various markets including treasury bills, IPOs, environmental permits, electricity, and spectrum licenses. In contrast to single-unit auctions, bidders in multi-unit auctions are concerned with marginal costs rather than price, and compete in terms of both prices and quantities, making the strategy space and analysis more complex. Therefore, theorems and generalizations of single-unit auctions cannot be extended to multi-unit auctions (Vickery, 1961) [20]. This complexity has led to lower research in this area, and mixed empirical results, making the study of multi-unit auctions ambiguous and theories like revenue equivalency debatable. This paper thus aims to address the gap in this literature by carrying out an empirical analysis to provide evidence, ranking sealed-bid uniform and sealed-bid discriminatory price auctions in terms of their revenue, specifically in the market for fractional real estate investments.

The price paid in multiunit auctions can be determined by various price rules; including a uniform price auction, introduced by Milton Friedman (Friedman, 1960); discriminatory price auctions or pay-as-bid auctions' and Vickery auctions. The Treasury markets in countries like the USA, Argentina etc. along with most auctions in Electricity Markets, European Union Emissions Trading System (EU ETS) etc. are based on uniform price auctions. Whereas the less common discriminatory price auctions can be observed in the auctions of sovereign debt in countries like Bangladesh, France, Greece etc. and in advertising auctions, art auctions, procurement auctions etc. Currently, there exists no revenue or efficiency ranking of these payment methods primarily due to the existence of demand shading. Intuitively, discriminatory price auctions should yield more revenue as they enable buyers to be charged higher prices. However, this is not true as the demand schedule submitted for these auctions is often lower due to demand shading or demand reduction. This is caused due to winners' curse.

This refers to the tendency for the winning bid in an auction to exceed the intrinsic value or true worth of an item. In fact, most empirical studies show that uniform-price auction yield higher revenues.

Fractional real estate investment refers to the practice of purchasing a fraction or a share of a real estate property instead of buying the entire property. This allows individuals to invest in high-value real estate properties that may otherwise be out of their reach, by pooling their resources with other investors, which is similar to the stock market. Fractional real estate investment platforms have become increasingly popular in recent years, as they allow individuals to invest in real estate without the large capital requirements typically associated with purchasing a property outright. They also provide an opportunity for diversification, as investors can spread their investments across multiple properties or real estate projects. This is a new and emerging method investment which can be done on various platforms like lofty AI, Fundraise, Cadre, Roofstock etc. Most of them currently use the post-it pricing method, i.e they provide a single price at which the buyer can either buy or not buy. Therefore, it's extremely restrictive. Many papers like Wang (1998) [21] state that an auction generates a higher revenue in most settings. Therefore, suggesting the need for these platforms to switch to auctioning the shares.

This paper contributes to the current literature by providing empirical evidence for the revenue ranking between these pricing methods to determine which payment strategy would help maximise the sales revenue as that is likely to be the main objective of the seller. Hence, current paper strives to contribute to the current theme in two ways. Firstly, it endeavours to reveals the fact related to quantity of biding under discriminatory and uniform price auction. Secondly, this paper strives to quantify the total amount of revenue generated under two different auction schemes. We are sanguine that the outcome of current research will play an essential role for the policy making pertaining to the revenue maximization of the firms. On the other hand, the outcome will also be beneficial in terms of understanding the behaviour of the bidder in different price auction. The paper is divided into 4 sections. Section 2 (Literature review) provides an overarching view of the empirical and theoretical studies done on multi-unit auctions, section 3 (Data and methodology) presents the way the research was carried out and some assumptions made, section 4 presents results, while section 5 concludes the study.

2. Literature Review

The literature on multi-unit auctions provides a mixed overview of the revenue ranking of the two auction types. First, I would provide some papers exploring and trying to provide theoretical proof of revenue equivalency in multi-unit auctions under stringent conditions. Then, some empirical evidence of the same is provided. Moreover, theoretical proofs giving a definitive revenue ranking under different settings and assumptions and empirical evidence of both uniform and discriminatory price auctions being dominant in terms of revenue is also provided.

Vickery (1961) [20] provided proof for the revenue equivalency theorem for single-unit auctions. This states that all auction types produce the same expected revenue under some assumptions like all bidders are rational, risk-neutral, symmetric and have independent private values. This was extended by Wilson (1979) [15] to multi-unit auction theory. He states that for every equilibrium bidding strategy in a uniform price auction, there is always a corresponding discriminatory price auction equilibrium. He also provides theoretical proof behind bid shading. Hummel (2018) [23] analyses a market with a single seller who wishes to sell a continuum of units of a single good to a continuum of bidders who have private values for the goods. He shows that in that setting, the post-it price method leads to the least profit highlighting the need of auctioneers, including, fractional real estate investment platforms to deviate from it. It also shows that there is no difference in the profits of both, discriminatory price auction and uniform price auction methods. Kylewood (1989) also establishes the revenue equivalency theorem in multi-unit auctions under some assumptions by stating that the expected revenues of discriminatory price auctions and uniform price auctions are the same. This is because the bid schedule in discriminatory price auctions may be lower due to demand shading and lower participation rate. This is caused due to the higher risk of being subjected to the winner's curse in such auctions. In addition, Cumpston (2020) [22] demonstrates that both these price mechanisms are inefficient. Highest-value bidders have a higher influence over the clearing price and therefore, have a higher incentive to shade their bids. This leads to a higher chance of them not getting the auctioned item leading to inefficient allocation. Cumpston also builds a model that leads to the revenue ranking of the two auction systems being ambiguous and an area for empirical research.

However, in general, multiple research and proofs do provide a revenue ranking. Ausubel (2004) [1] proves that in cases of symmetric private values amongst bidders, all equilibria attained by discriminatory price auctions can and will dominate the equilibrium of uniform price auctions. This thus, goes against the revenue equivalency theorem. Moreover, Wang and Zender (2002) [18] model the US treasury securities auction in order to present the results, however, these results are applicable more generally as well. They prove that if bidders are symmetrically informed and risk-neutral, then discriminatory pricing leads to extremely high competition, and thus, the bidders submit a flat bid schedule. They theoretically prove that with symmetrically informed risk neutral competitive bidders; sellers expected revenue in almost all equilibria of a uniform price auction is less than in the unique equilibrium of a discriminatory auction. Moreover, even though risk-averse bidders compete less aggressively, their discriminatory price equilibria still exceed the equilibrium of uniform pricing. The paper proves that when bidders are symmetrically informed, risk averse and competitive; the seller's expected revenue in a discriminatory auction is strictly greater than the expected revenue from the bidder’s most preferred equilibrium of a uniform price auction. This implies that when the bidders are symmetrically informed, there always exists equilibria of a uniform price auction with lower expected revenue than a discriminatory price auction. Moreover, Ausubel, *et al.* (2014) [2] concludes that in symmetric private value auctions, pay-as-you-bid auctions often outperform uniform price auctions in terms of sales revenue and efficiency. But, relaxing the assumptions of risk neutrality and symmetry results in uniform-price auctions outperforming the discriminatory payment format. Uniform price auctions have other benefits too, including but not limited to their high. These advantages and disadvantages thus, make it difficult to theoretically outline which price rule is superior in terms of revenue or efficiency. Pekec and Tsetlin (2008) [14] conclude that if the uncertainty about the number of bidders is substantial then, discriminatory price auctions yield higher revenues than uniform price auction. Also, Sushil Bikhchandani and Chi-fu Huang (1939) [3] create a model of competitive bidding with a resale market. Here, the primary auction is a common value auction wherein bidders participate with the intention to resale in the secondary market. In the secondary market, buyers get information about the bids of the primary auction. The paper studies this information linkage between this secondary and primary market and using an exploratory model of treasury bill markets. In this setting, there exists two type of bidders: competitive and non-competitive. Non-competitive bidders bid almost always win their requested quantity while the Competitive bidders (Around 40 large financial institutes who are risk neutral and have private information about the true value of the objects.) compete for the remaining. Summary statistics are then released before the trading on the secondary market can begin. Secondary bidders have access to these primary market bids which is very important as it reveals private information about competitive bidders; which has great influence. This thus incentivises primary market bidders, especially in a uniform price auction to submit extremely high bids to deceive secondary market bidders. The paper thus concludes that the auctioneers expected revenue from organising a uniform price auction (Bidders pay the highest losing bid) in the primary market would be more than if the first price discriminatory action was implemented. However, all these theoretical results differ greatly, are ambiguous, or are true only under strict, impractical considerations. Thus, making it vital to consider empirical studies in order to make any generalisations.

The outcome of most empirical studies also differs greatly. Kang and Pullers' (2008) [24] study on the Korean treasury market provides empirical evidence of discriminatory auctions outperforming uniform price auctions, despite the fact that both are extremely competitive. It proves discriminatory auctions (winner bidders paying the price bid for each unit purchased)leads to higher revenue than the upper bound of Vickery auctions while uniform auctions(the winning bidders paying the market clearing price for all units purchased) revenues are always below the upper bound for Vickery auctions. However, these differences are really small. It also states that both these pricing strategies are inefficient compared to Vickery. But at the same time, discriminatory price auctions are more efficient (If the set of bidders is constant) in allocating the Treasury Bonds to high-value bidders. Moreover, as the number of bidders grows larger, these auctions become more and more efficient. This is because participants tend to bid their true value in uniform price auctions as the number of bidders increases resulting in a reduction in their market power. Moreover, the research on the Zambian foreign exchange market conducted by Rafael Tenorio (1993) [16] tries to determine if the change in the format of the auction has an impact on the bids and revenue. Despite controlling many variables and having the same setting, empirical evidence suggests that the competitive format yields a higher average revenue than the discriminatory format. A major reason of this could be due to the public information in forming exchange rate forecasts or the higher number of participants under this format perhaps due to the ease to decide upon a strategy. Feldman and Reinhart (1995) [8] analysed the data from an IMF gold auction run in 1976-1980. The evidence suggests that the revenue generated in uniform price auctions are higher than the discriminatory auction. Thus, concluding that the magnitude demand shading in discriminatory auctions exceeds the optimum demand shading that leads to revenue equivalency. Moreover, Heller and Lengwiler (2001) [9], Mishra (2021) [13] use the real data from uniform price Treasury bond auctions in Switzerland to generate counterfactual data for discriminatory auctions. Using this, the paper concludes that uniform price auctions generate higher revenue. Thus, this disparity between various theoretical works and empirical evidence makes this an interesting topic of study for the research paper.

3. Data and Methodology

3.1 Data

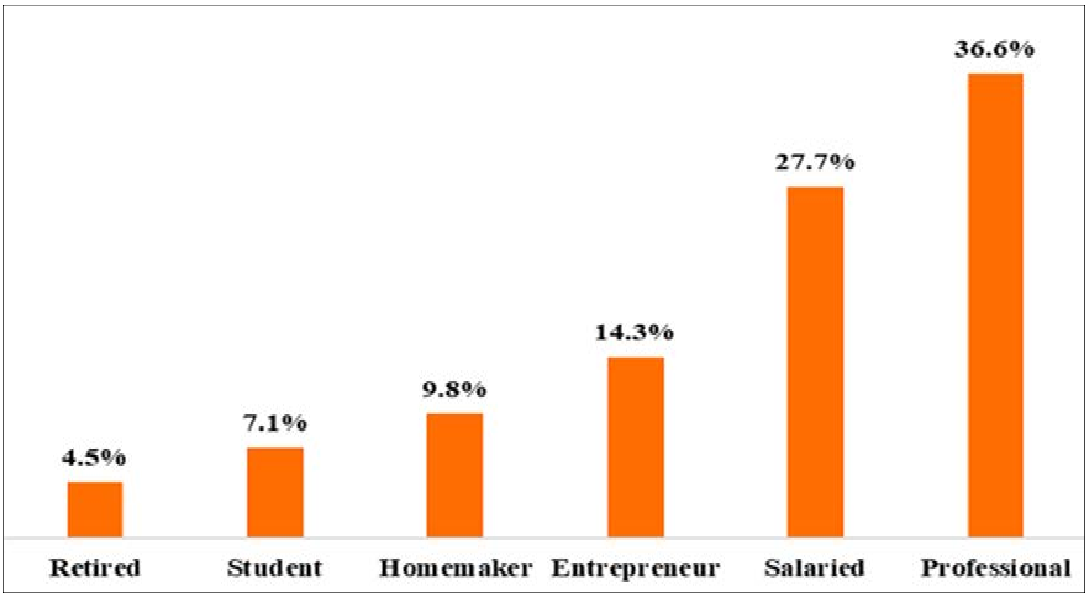
Data collection

The information for this study is gathered from primary sources. This type of study necessitates a detailed questionnaire type of observation and analysis; hence a 12-question questionnaire with various options was created. The data presented was collected through primary research across a sample of 112 respondents. For collection of data, this research relied on the Google form, the diagrammatic and descriptive statistics have been presented below.

Sample size

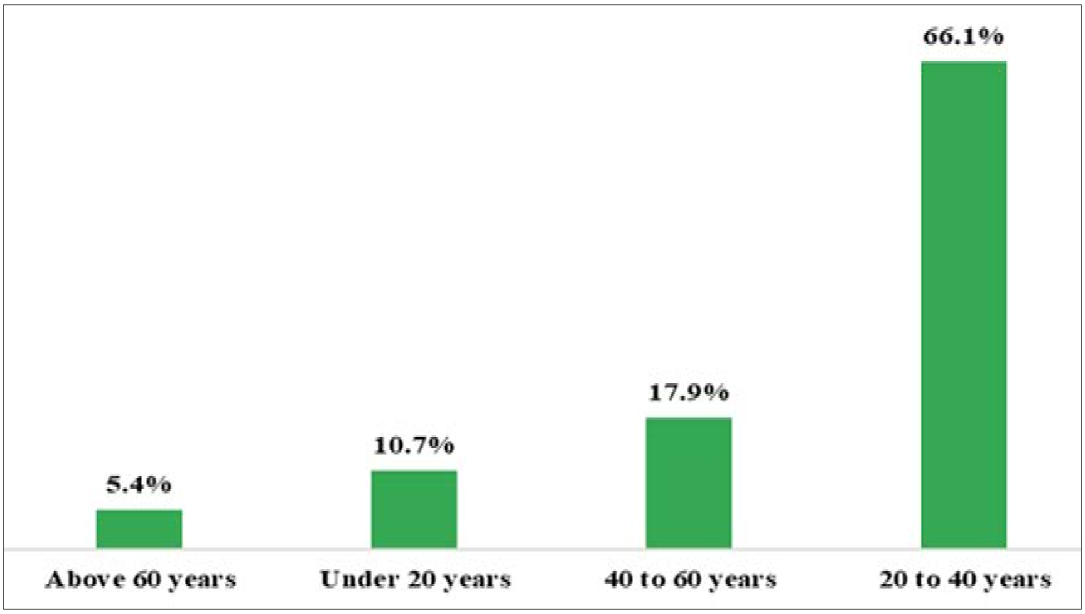
The entire sample size of total respondents are 112.

Questionnaires was distributed randomly and the sample for this study have been selected from the data received from a survey questionnaire that current paper conducts in region of Delhi NCR. The graph shows that the highest percentage (36.6%) of the respondents were professionals, followed by salaried workers and Entrepreneurs highlighting the credibility of the study. Only a few 7.1% were students.



Source: Author Calculations

Fig 1: Percentage of respondents based on profession



Source: Author Calculations

Fig 2: Percentage of respondents based on age

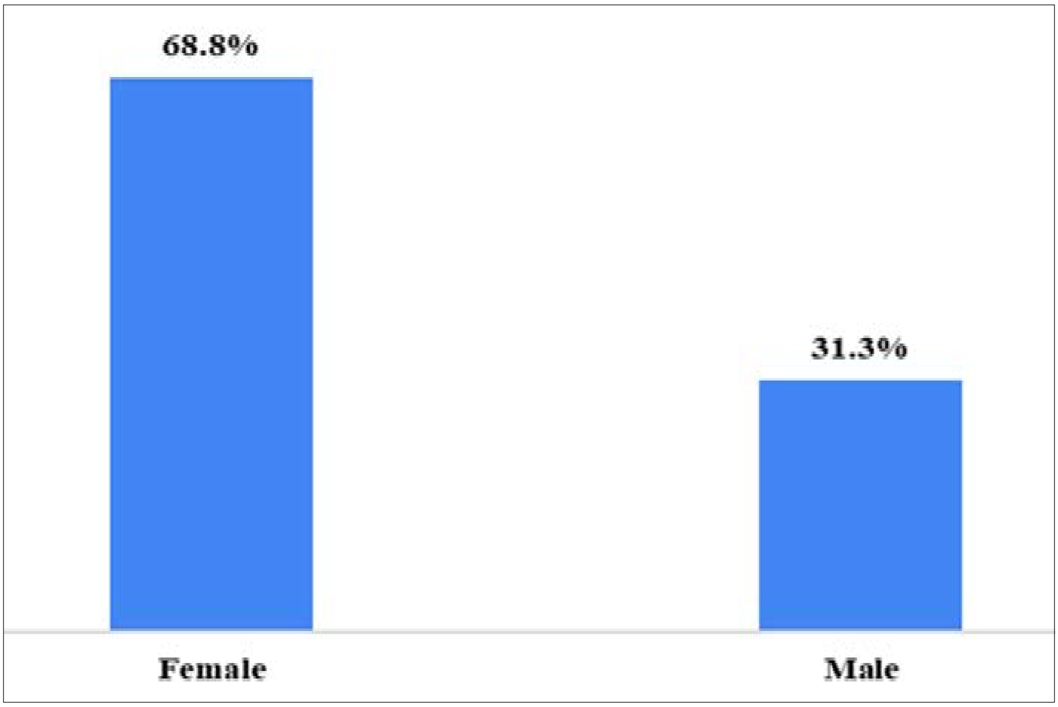
68.8% of the respondents were female and 31.3% were male.

Table 1: Statistics comparing the two Auctions bidding quantity

Statistics	Discriminatory Auction	Uniform Auction
Mean	2.366071	3.785714
Median	2.000000	4.000000
Maximum	5.000000	5.000000
Minimum	1.000000	1.000000
Std. Dev.	0.859452	0.953217
Skewness	0.847614	-0.564852
Kurtosis	3.631872	2.726609
Jarque-Bera	15.27427	6.304551
Probability	0.000482	0.042755
Sum	265.0000	424.0000
Sum Sq. Dev.	81.99107	100.8571
Observations	112	112

Source: Author Calculations

66.1% of the respondents were aged between 20 and 40 and 17.9% were between 20 to 40 years old. Only 5.4% were above 60 years old.



Source: Author Calculation

Fig 3: Percentage of respondents based on gender

We begin our analysis with the presentation of descriptive statistics of quantity of biding under discriminatory and uniform price auction scheme. Table 1 demonstrates that the average biding under discriminatory is lesser than the uniform price auction. The result also reveals that, the biding under uniform auction seems more volatile as compare to discriminatory auction as documented by the outcome of standard deviation. The descriptive statistics of biding under uniform auction have negative skewness, while discriminatory price auction is positively skewed. The assumption of normality is decisively precluded for both the price auction, as the assumption of normality is definitely rejected for both the price auction according to the Jarque-Bera normality test.

3.2 Methodology: The paper analyses their revenue ranking in the context of a new and emerging investment market: the fractional real estate investment market. Here, each share is homogenous and they are not complimentary or substitute to each other. Thus, this helps analyse one of the simplest forms of multi-unit auctions. The paper uses Lofty AI, specifically, the paper chose 621 E Le Claire Rd as the property whose share's auction was analysed. A survey was made in order to assess the revenues generated by both these auction formats for the auction. It provided the bidders with full and symmetric information about the properties, their financials, etc. The helped assess their demand schedule for different auction settings as well as a real demand schedule. As this is just a hypothetical situation and does not have monetary repercussions for the bidders; they would have no incentive to lie about their real demand curve. These individual demand schedules were aggregated in order to form a real market demand curve and market demand curve for uniform and discriminatory price auctions. This was done by adding aggregating the Quantity demanded at each price and plotting those points on a graph, then forming the equation of a best fit polynomial regression up to 2 degrees.

Research Design

Descriptive research design: Descriptive design is a scientific method that involves looking at and describing the behaviour of the bidder under different auction scheme.

Hypothesis Testing: Based on the above literature, following hypothesis have been formed

H_0 : There is no difference in average bidding quantity under discriminatory and uniform price auction scheme

H_1 : There is no difference in variability of bidding quantity under discriminatory and uniform price auction scheme

Regression Model

$$price = \alpha + \beta(quantity) + \delta(quantity)^2 + \epsilon$$

We have applied above regression model to develop the demand curve on the data that has been gathered through the questionnaire. The demand curve is further applied to calculate the total revenue under the different auction scheme.

T Test for mean difference of quantity bidding under different scheme

The t test estimates the true difference between two group means using the ratio of the difference in group means over the pooled standard error of both groups.

$$t = \frac{QD_d - QD_u}{\sqrt{S^2 \frac{1}{n_1} + \frac{1}{n_2}}}$$

In the above equation, t is the t value, QD_d and QD_u are the average quantity of bidding under the discriminatory and the uniform price auction. On the other hand, S^2 is the pooled standard error of the two groups, and n_1 and n_2 are the number of observations in each of the groups. In the above test process, our null hypothesis states that there is no difference in biding quantity under the discriminatory and the uniform price auction. On the other hand, alternative hypothesis assumes the significance differences in the biding quantity under both scheme of biding. A larger t value shows that the difference between group means is greater than the pooled standard error, indicating a more significant difference between the groups.

3.3 Assumption

1. The auction settings were multi-unit, static, and sealed bid as the survey was filled by bidders in isolation without conference with other bidders or information about other bids.
2. Informationally symmetric as the same information was given to all bidders suggesting the real value of each share. This led to the share having a shared common value in the auction.
3. It is fair to assume that no collusion takes place as the identity and bids of each bidder are not shared
4. Respondents are rational with the aim to maximise their payoff. Considering that most participants in these auctions do their due diligence, they are assumed to choose a bid which that maximises their expected payoff
5. St. Petersburg game was used to determine that each bidder was risk averse. In this game, a fair coin is tossed at each stage. The initial stake is $2 and doubles each time head appears. But the game ends when tail appears. The expected payoff could be calculated with the expected value formula: with a probability of ½ that the player wins $2; with a probability of ¼ that the player wins $4 and so on. Upon calculating the series, we can determine that and thus $E = \infty$. Bidders were asked the maximum price they were willing to pay to participate in this game. Their answers were all below infinity, so it was concluded that bidders are willing to pay less than the expected value for a risky game and are hence risk averse.
6. We also reduced the number of shares available to 1237 in order to adjust it with the lower number of participants as compared to the real auction.

4. Result Analysis

4.1 Result of quantity biding in different price auction

For testing the significant differences on the average quantity of bidding under the discriminatory and the uniform price auction, we exert student's t test. The student's t test gives accurate outcomes in case when we do not have any information about the true population variances. The outcomes of t test reveals that we can discard the assumption of equality average quantity of bidding under the discriminatory and the uniform price auction, at less than 5 percent level of significance. Our result clearly reveals that investors do react differently under discriminatory and the uniform price auction, hence it can be surmised that nature of price auction does have significant impact on the average quantity of share people buy with special reference with Indian financial market. Our result exhibits the persistence of difference in the biding quantity under the different price schemes, however there might be question arises why it is so. The answer to the current question can be explained with many reasons.

On the other hand, to examine the significant differences in the variability in biding quantity between the discriminatory and the uniform price auction, we apply F test. The F test gives accurate outcomes in the case when we deal with variances. The outcomes of F test reveals that we can reject the null hypothesis of equality in variance of biding quantity under two price schemes, at less than 5 percent level of significance. These outcome of t and F test clearly justify the difference in biding quantity under discriminatory and the uniform price auction in Indian context.

Table 2: Outcome of t and F Test for Equality of Means and variance between two auctions

Method	Value	Probability
T-test	-11.70588	0.0000
Satterthwaite-Welch t-test*	-11.70588	0.0000
Anova F-test	137.0277	0.0000
Welch F-test*	137.0277	0.0000

Source: Author calculations

4.2 Regression outcome under different bidding scheme

Table 3: Outcome of Regression for discriminatory auction

Coefficients	Value	t statistics
Intercept	8057.7	5.36***
Quantity	-2.614	-3.85***
$(\text{Quantity})^2$	0.0002	2.65**
R Square	0.97	

Source: Author calculations
Note: *** is significant at 1% and ** at 5%

Table 4: Outcome of Regression for Uniform auction

Coefficients	Value	T Statistics
Intercept	5556	5.36***
Quantity	-1.420	-4.85***
$(\text{Quantity})^2$	0.0002	2.89**
R Square	0.92	

Source: Author calculations
Note: *** is significant at 1% and ** at 5%

4.3 Result of Revenue in different price auction

The individual quantity demanded (QD) by individuals was aggregated in order to determine the total QD at each price. This helped determine the market demand schedule which was plotted on a graph to obtain the equations for different demand curves.

Table 5: Total Quantity (QD) at every price for each auction type

Price	QD in uniform price auction	QD in Discriminatory price auction	QD in the real Demand
3700	2038	1662	2213
4100	1817	1194	2032
4500	1587	837	1855
4900	1372	502	1684
5351	1159	142	1468

Source: Authors calculations

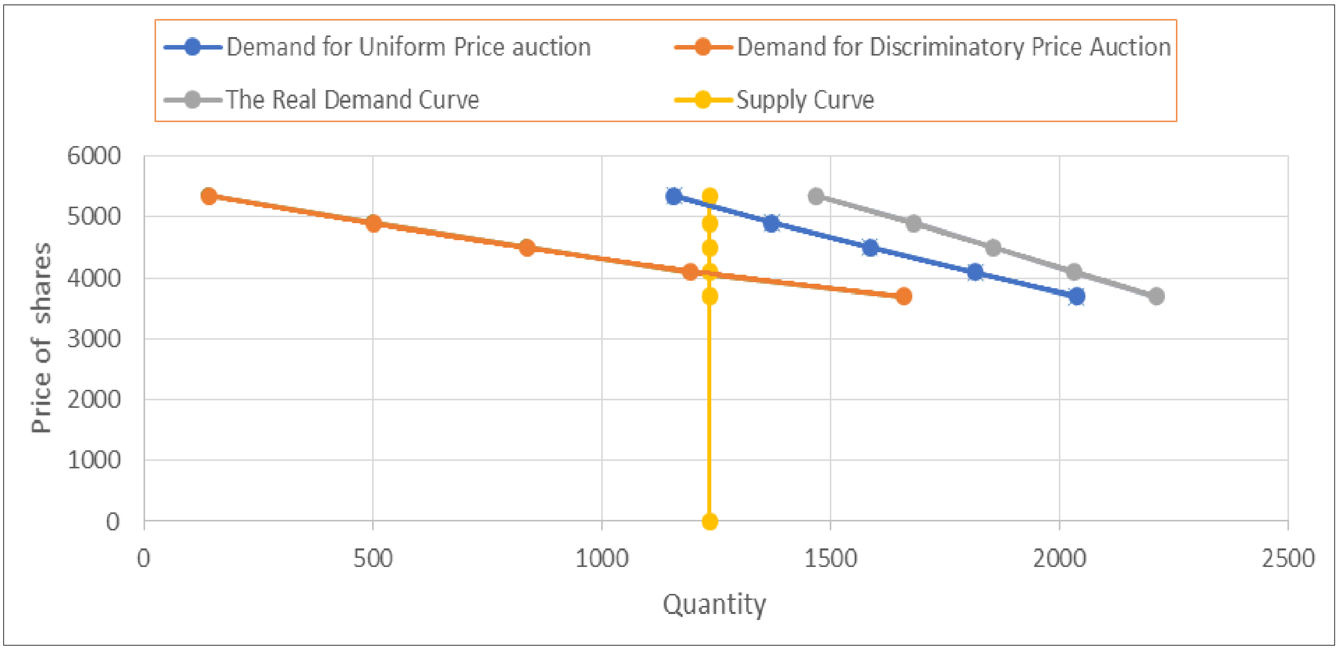


Fig 5: Total demand curves

Total Revenue under the uniform price auction

The equation for the demand curve in a uniform price auction is $P = 0.0002q^2 - 2.6148q + 8057.7$ with the r-squared value of 0.9997. The clearing price, or P when Q=1237 is Rs 5129.2262. As in uniform price auctions; all bidders are expected to pay the same price the total revenue (TR) can be calculated to be $TR = P \times Q$. So the $revenue = 1237x5129.2262 = \text{Rs. } 6344852.81$

Total Revenue under the discriminatory price auction

The equation for the demand curve in a discriminatory price auction is $P = 0.0002q^2 - 1.4204q + 5556$. With the r-squared value of 0.9997. The clearing price is Rs.4104.999. As all bidders pay what they bid, the total revenue would be the area under the curve from the origin to the quantity traded, which is 1237.

Thus,

$$revenue = \int_0^{1237} 0.0002q^2 - 1.4204q + 5556 \, dq = Rs. 5077883.76$$

Total Revenue under the real value price auction

The equation for the real demand curve was found to be P= $-0.00009q^2 - 1.8905q + 8330.6$ with the r-squared value being 0.9998. If the bidders bid their true values, the clearing price would be approximately Rs. 5854.33629

In discriminatory price auctions, share demand is lower than in uniform price auctions, which is in turn lower than the real demand curve. The demand curve gradient for discriminatory price auctions is lower than both the real valuation and the demand for uniform price auctions. Additionally, the revenue earned in discriminatory price auctions is only lower by Rs. 1266969.1 or 19.96845%.

The demand is lower in discriminatory price auctions due to their complexity, difficulty in finding equilibrium, difficulty in strategy development and the perception of unfairness associated with different prices for the same units. In these auctions, bidding real values would lead to zero payoffs. Thus bidders bid cautiously to maximize their consumer surplus or profit, leading to a trade-off between profit and the chance of receiving the good or service. The fear of the winner's curse, where the winning bidder may overpay due to overestimating the worth, also contributes to lower demand. These factors result in high demand shading in the discriminatory price auction, which ultimately leads to lower sales revenue compared to the uniform price auction.

In discriminatory price auctions, the demand curve is flatter and more price elastic. Bidders are sensitive to price changes because they know that others may be receiving different prices for the same item. This leads to a higher likelihood of bidders walking away from the auction if they feel the price is too high relative to their perceived value, considering that others may be receiving better prices, and vice versa.

The market demand curve in uniform price auctions is also lower than the real demand curve. Binmore and Swierzbinski (2000) [4] suggest that the "fog of war" may contribute to this phenomenon. This refers to the dangers of other players may not act rationally, such as by bidding unreasonably high amounts. This behaviour can make other participants more cautious in their bidding, leading to higher market uncertainty and demand shading. The demand may also be influenced by the winners curve, but to a lesser extent as bidder would end up paying the same price reducing the chance of them overpaying. Also, high-valuation bidders with significant market power may also shade their bids to lower the clearing price. The demand is also relatively inelastic. Ausubel and Cramton (2002) [25] argue that after the first bid is made, every additional bid increases the expected price to pay with earlier own bids.

This means that at a minimal quantity, the two curves intersect at the same price, but at higher quantities, the bidden price for additional units will be lower than their true value. This steep bid curve can result in inefficiency as smaller bidders may end up purchasing goods that have a lower value relative to larger bidders.

However, the revenues generated by the two auctions in this paper differ by an extremely small percentage which may be statistically insignificant. This may prove the revenue equivalency by Vickery (1961) [20] in a multi-unit auction. While most literature supporting revenue, equivalence relies on strict assumptions like independent private values, this paper extends the conclusion to a setting of common value for the goods. Additionally, it aligns with the findings of other studies by Wilson (1979) [15] and Cumpston (2020) [22], Mishra (2019a) [12] which suggest that the revenue ranking is ambiguous. It also supports papers by Tenorio, Feldman, and Reinhart (1995) [8] and Heller and Lengwiler (2001) [9], Mishra (2019) [11] demonstrating that uniform price auctions yield higher revenue. Moreover, the extension of these findings to a different setting and market of fractional real estate investments indicates broader applicability beyond specific assumptions. However, it is important to note that the small difference in revenues observed may be a random outcome.

5. Conclusion

In conclusion, the paper provides empirical evidence suggesting that uniform price, multi-unit auctions yield a slightly higher revenue as compared to discriminatory price multi-unit auctions, particularly in the context of fractional real estate investment. While the study focused on a specific industry (fractional real estate investment), the findings may have implications for other industries that use multi-unit auctions, such as commodity markets or government procurement auctions. The use of empirical evidence in this study helps validate theoretical predictions giving practical insights for auctioneers and policymakers. This implies that implementing uniform price auctions may be beneficial for auctioneers, like fractional real estate platforms, including LOFTY AI. They generate higher revenue, have higher participation and are more allocative efficient. This proves that many auctions like Treasury auctions, Electricity Markets, European Union Emissions Trading System (EU ETS), etc. use the higher revenue-generating auction formats. This also suggests that discriminatory price auctions should change their format proving Milton Friedman opinion right. However, this may make a minimal difference as the difference in revenues is extremely low. Thus, the less common discriminatory auctions might also generate an almost equivalent revenue depending on factors. The result is consistent with the previous research as it suggests that the uniform price auctions may yield a higher revenue than discriminatory price auctions. However, the difference is extremely small which may mean that it is statistically insignificant. Moreover, there are many limitations to the study. For example, the small sample size suggests that future research is needed to confirm the findings and explore additional factors that may impact auction revenue. Also, the degree of risk aversion may differ amongst the respondents. Also, each respondent could have it may not be in the capacity or best interest of the respondents to analyse all the data and respond rationally, considering that there was no monetary incentive or cost for them if they won the auction. There may be sampling bias too and the respondents might not have been representative of the entire population. Lastly, every respondent may not have the same tools, experience, etc. to analyse the data given in order to derive the same common value and the optimum strategy with the maximum payoff for them.

Also, the results may be specific to the setting and may not be generalizable to other contexts; especially considering that the difference is extremely minimal. Therefore, it is recommended to conduct additional empirical studies in different industries and settings to generalize the findings and identify industry-specific factors that impact the revenue ranking of auction formats. Exploring the impact of different auction design elements such as reserve prices, minimum bid increments, choice of auction platform, bidder qualification criteria, and auction timing etc., and changing information symmetry, risk-aversion of bidders, and the existence of a secondary market can help identify specific strategies to optimize auction revenue. It is also crucial to compare the revenue ranking of these auctions over time to identify any trends or changes in the factors that influence auction outcomes. Lastly, conducting a meta-analysis of studies that have compared the revenue ranking of discriminatory and uniform price auctions could provide a more comprehensive understanding of the factors that impact auction outcomes. Overall, these additional studies can enhance the understanding of auction mechanisms and lead to improved auction design and revenue generation in different industries.

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